# Rapidly rotating black holes in Chern-Simons gravity: Decoupling limit solutions and breakdown

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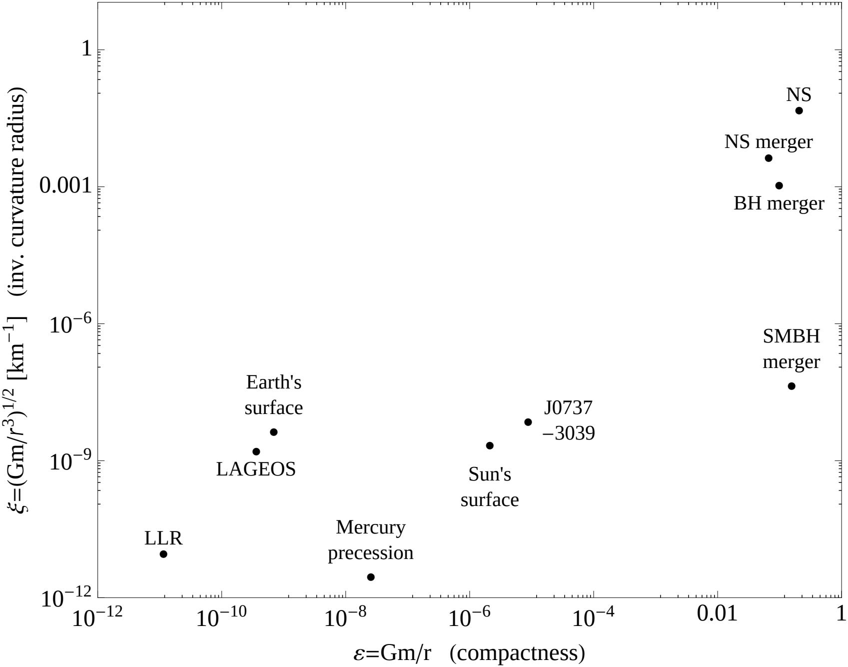
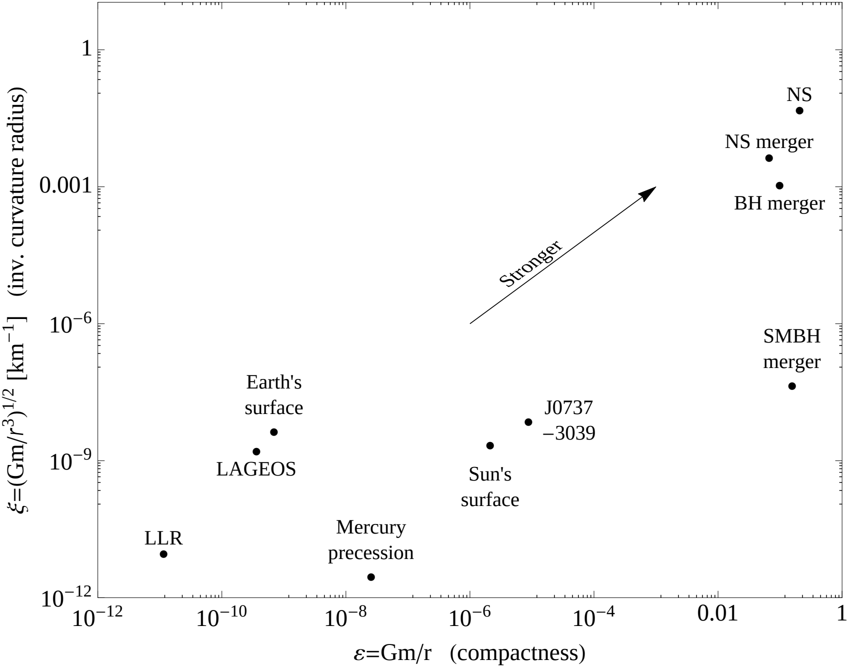
# Motivation

1. GR successful but incomplete
   - GR+QM=new physics (e.g. BH thermo)
   - Planck scale phenomena? Other scales?
   - Expect GR is low-energy EFT
2. Ask nature
   - So far, only weak-field tests
   - Lots of theories ≈ GR
   - Need to explore strong-field
     - Strong curvature • non-linear



NS
NS merger
BH merger
SMBH
merger
Earth's
surface
LAGEOS
J0737
−3039
Sun's
surface
LLR
Mercury
precession
1
0.001
$10^{-6}$
$10^{-9}$
$10^{-12}$
$10^{-12}$
$10^{-10}$
$10^{-8}$
$10^{-6}$
$10^{-4}$
0.01
1
$\xi$=(Gm/$r^3$)$^{1/2}$ [km$^{-1}$] (inv. curvature radius)
$\varepsilon$=Gm/r (compactness)



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What phenomena come from UV completions?

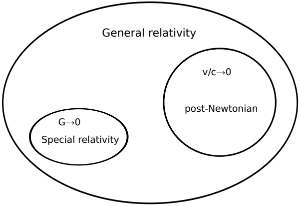
# Theories

Fundamental approach:

- String theory • Loop quantum gravity
- TeVeS • Einstein-Æther • Hŏrava
- Massive gravity • dRGT • bi-metric
- . . .

Pedestrian approach: effective field theory

- Learned from cond-mat, then nuclear and hep-th
- Theory with separation of scales
- "Integrate out," effective theory for long (or short) wavelengths
- Works backwards!



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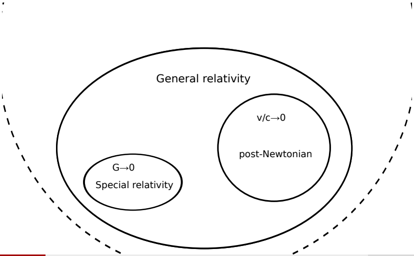


# EFT works

Worked for describing superconductivity, predicting W, higgs, etc.

Try to build EFT for gravity

- Metric, general covariance, Lorentz invariance
- Lowest order dynamical theory is $\Lambda$+GR!

Beyond GR: add new $\ell$—want to constrain this

# Dynamical Chern-Simons Gravity

$$S = \int d^4x \sqrt{-g} \left[ \frac{m_{\rm pl}^2}{2} R - \frac{1}{2} (\partial\vartheta)^2 + \frac{m_{\rm pl}}{8} \ell^2 \vartheta \, {}^*RR \right]$$

- Anomaly cancellation, low-E string theory, ...
- Lowest-order EFT with parity-odd $\vartheta$, shift symmetry (long range)
- Phenomenology unique from other $R^2$ (e.g. Einstein-dilaton-Gauss-Bonnet)
- Tractable
- Straw-man theory

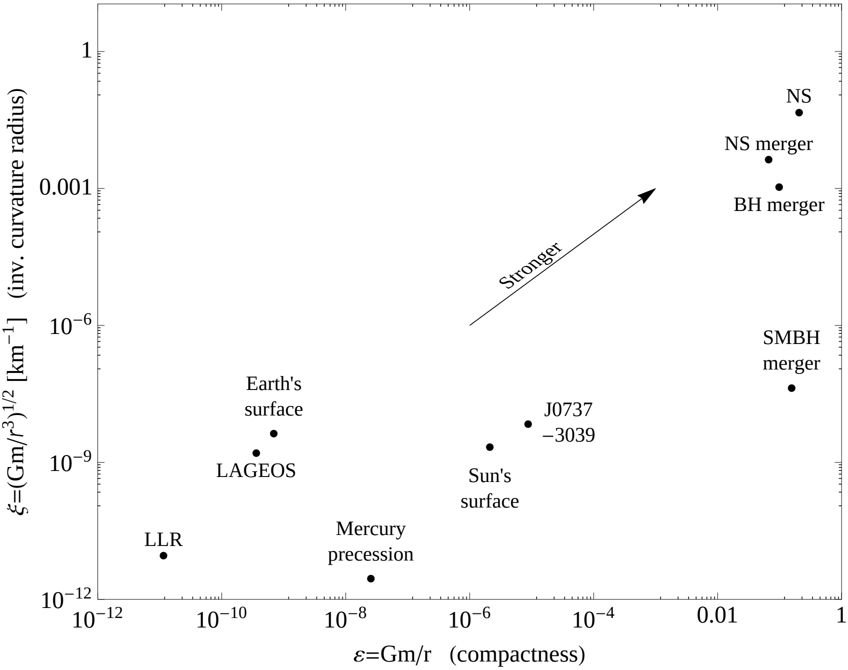
# Decoupling limit

- Most EFTs don't make sense as exact theories (see e.g. Delsate+Hilditch+Witek)
- (Almost) All corrections introduce new $\ell$
- Can't be too long
- Expand fields, EOMs in powers of $\ell/\mathcal{R}_{\rm BG}$, perturbation scheme

Question: What is regime of validity of decoupling limit?

# How do we constrain $\ell$ in dCS from astronomical observations?

- dCS is higher-curvature and parity-odd



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# How do we constrain $\ell$ in dCS from astronomical observations?

- dCS is higher-curvature and parity-odd

$$\Box\vartheta = -\frac{m_{\rm pl}}{8}\ell^2\ {}^*RR\,,$$

- Want ${}^*RR$ as large as possible
  $\Longrightarrow$ smallest $M$, largest $\chi = J/M^2$
- NSs have small M, but BHs have $\chi \to 1$

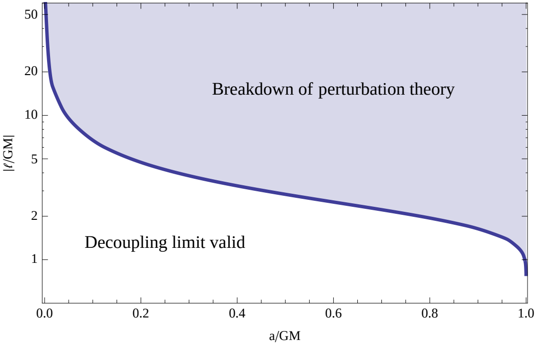
Want solutions for rapidly rotating black holes in dCS

# Black holes in dCS

- $a = 0$ (Schwarzschild) is exact solution with $\vartheta = 0$
- Analytically known solutions in decoupling limit
  - $a \ll M$ limit up to $\mathcal{O}(a^2)$, valid $\forall r$ (see Yunes+Pretorius, Yagi+Yunes+Tanaka)
  - $r \gg M$ limit for $l = 1$, valid $\forall a$ (see Yagi+Yunes+Tanaka)
- I construct numerical solutions $\forall r, \forall a$

# Takeaway

- I construct numerical solutions $\forall r, \forall a$ for dCS BHs in decoupling limit
- I use solutions to determine the regime of validity of PT




- This can be turned around to forecast bounds $\ell \lesssim 22\text{km}$ from GRO J1655–40 ($M = 6.30 \pm 0.27 M_{\odot}$, $\tilde{a} \approx 0.65$–$0.75$)
- For details see Phys. Rev. D 90, 044061 (2014) [arXiv:1407.2350]

# Equations to solve

$$S = \int d^4x \sqrt{-g} \left[ \frac{m_{\rm pl}^2}{2} R - \frac{1}{2}(\partial\vartheta)^2 + \frac{m_{\rm pl}}{8} \ell^2 \vartheta \, {}^*RR \right]$$

$$\Box\vartheta = -\frac{m_{\rm pl}}{8} \ell^2 \, {}^*RR$$

$$m_{\rm pl}^2 G_{ab} + m_{\rm pl} \ell^2 C_{ab} = T_{ab}^{(m)} + T_{ab}^{(\vartheta)}$$

- $g^{ab} C_{ab} = 0$

# Decoupling limit

Take $\ell^2 \to \varepsilon \ell^2$ and expand in $\varepsilon$,

$$\vartheta = 0 + \varepsilon \vartheta^{(1)} + \frac{\varepsilon^2}{2} \vartheta^{(2)} + \dots$$
$$g = g_{\mathrm{GR}} + \varepsilon 0 + \varepsilon^2 h^{\mathrm{def}} + \dots$$
$$\Box_{\mathrm{GR}} \vartheta^{(1)} = -\frac{m_{\mathrm{pl}}}{8} \ell^2 \; {}^{*}RR[g_{\mathrm{GR}}]$$

Don't yet have quantity to test validity of perturbation theory

# Next order

$$m_{\rm pl}^2 G_{ab}^{(1)}[h^{\rm def}] = T_{ab}^{(\vartheta)}[\vartheta^{(1)}, \vartheta^{(1)}] - m_{\rm pl}\ell^2 C_{ab}[\vartheta^{(1)}]$$

- Trace equation
- Lorenz gauge $\nabla_a \bar{h}^{ab} = 0$

$$\frac{1}{2} m_{\rm pl}^2 \Box h^{\rm def} = -(\nabla^a \vartheta^{(1)})(\nabla_a \vartheta^{(1)}) \,,$$

- Same scalar PDE operator
- Caveat: gauge-dependent but should still capture $a$-dependence

# Next order

- Now can make comparison:

$$\sqrt{-g} = \sqrt{-g_{\mathrm{GR}}}\left(1 + \varepsilon^2 \tfrac{1}{2} h^{\mathrm{def}} + \mathcal{O}(\varepsilon^3)\right)$$

- If $h^{\mathrm{def}} \sim \mathcal{O}(1)$, should keep higher $\mathcal{O}(\varepsilon)$
- Criterion for validity of PT:

$$|h^{\mathrm{def}}| \lesssim 1 \qquad \text{everywhere}$$

- Program: Solve for $\vartheta^{(1)}, h^{\mathrm{def}}$ as functions of $r, \theta$ for all $a$

# Approach to solving

Symmetry reduced, $\vartheta = \vartheta(r, \theta)$. $\Box \to \Delta$.

Analytical:

- Static Green's function $\Delta^{-1}$ known analytically
- Separation of variables

$$\vartheta = \sum_j \vartheta_j(r) P_j(\cos\theta)$$

  - Can do source decomposition (See Konno+Takahashi and [arXiv:1407.0744])

Resort to numerics!

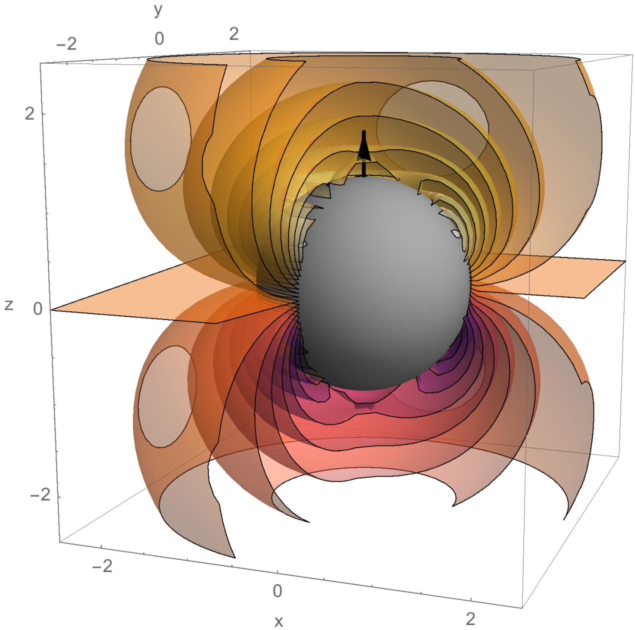
# Numerical approach

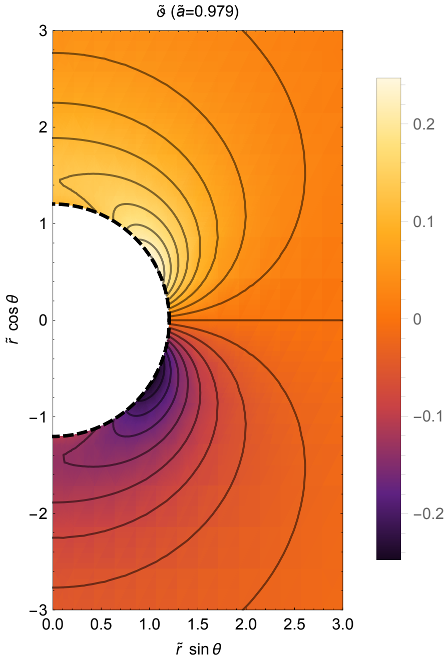
- Elliptic PDE. Could solve hyperbolic, parabolic, relaxation scheme
- Numerical separation of variables. Each $j$ mode is an ODE.
- Compactify $r$
- Pseudospectral collocation method
- Directly solve discrete ODE operator ("numerical Green's function")

# Numerical approach

- For each $a$, find $\vartheta(r,\theta;a)$, compute $(\partial\vartheta)^2$, find $h^{\mathrm{def}}(r,\theta;a)$
- Evaluate $\max|h^{\mathrm{def}}|$ and find regime of validity

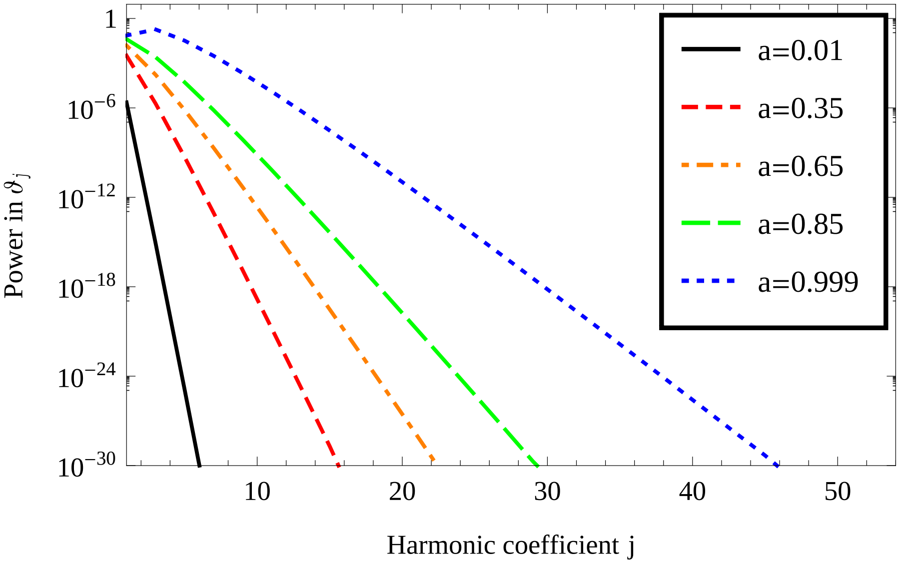
$\tilde{\vartheta}$ ($\tilde{a}$=0.979)





$\tilde{\vartheta}$ ($\tilde{a}$=0.979)
3
2
1
0
−1
−2
−3
$\tilde{r}$ cos $\theta$
0.0
0.5
1.0
1.5
2.0
2.5
3.0
$\tilde{r}$ sin $\theta$
0.2
0.1
0
−0.1
−0.2

# Exponential convergence



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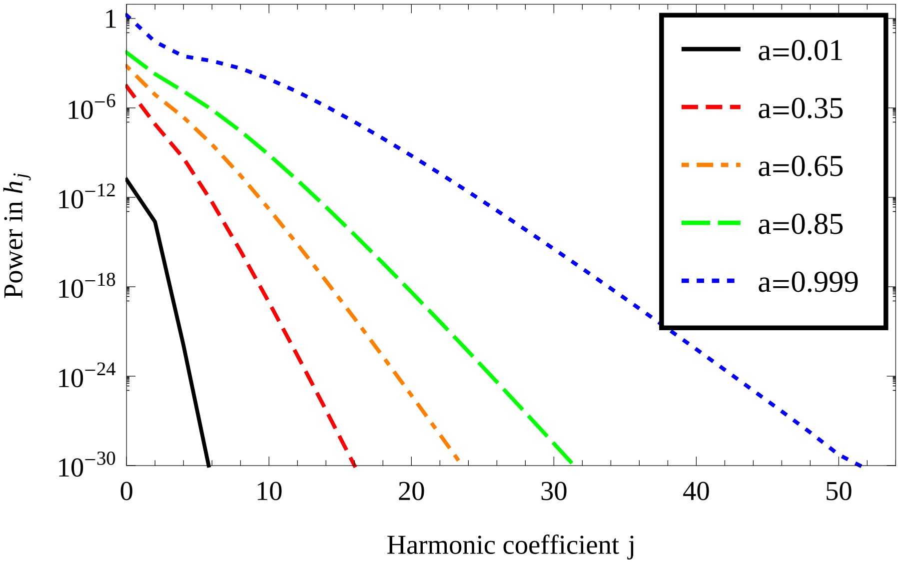

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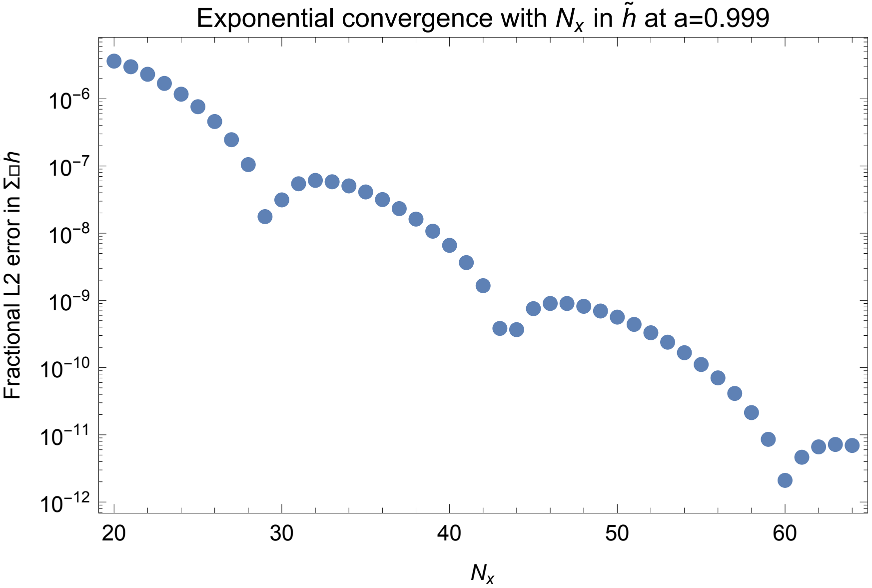
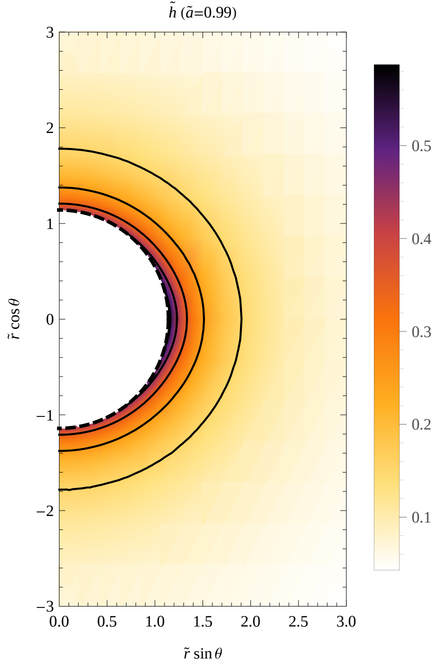
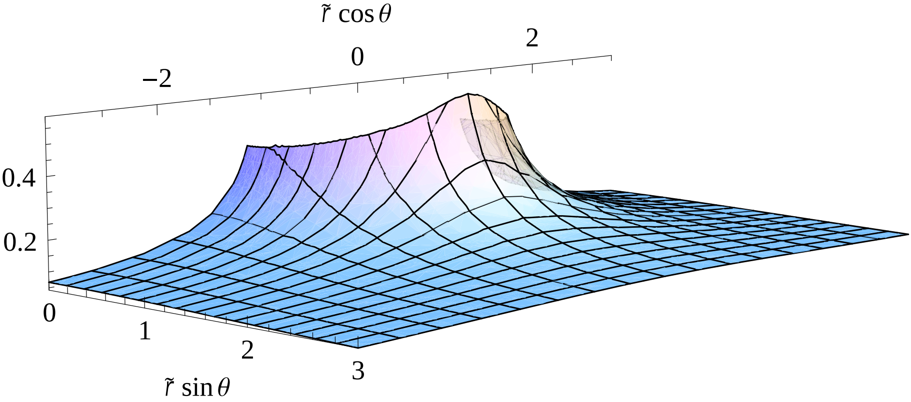
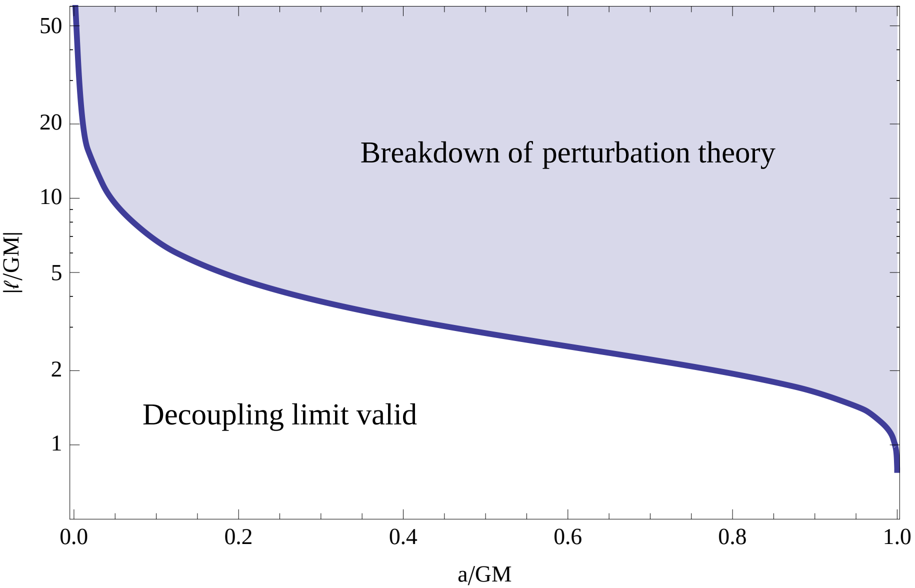




$\tilde{h}$ ($\tilde{a}$=0.99)
3
2
1
0
−1
−2
−3
0.0
0.5
1.0
1.5
2.0
2.5
3.0
$\tilde{r}\cos\theta$
$\tilde{r}\sin\theta$
0.5
0.4
0.3
0.2
0.1

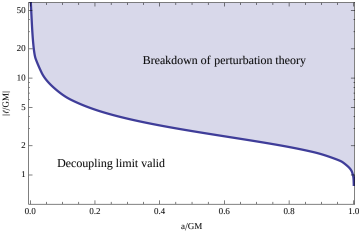

$\tilde{r}\cos\theta$
−2
0
2
0.4
0.2
0
1
2
3
$\tilde{r}\sin\theta$

# Regime of validity



# Forecasting bounds

- Observation of BH indistinguishable from GR predictions
- Size of $\ell$ correction below breakdown (caveat: cancellation)
- GRO J1655–40: $M = 6.30 \pm 0.27 M_{\odot}$, $\tilde{a} \approx 0.65$–$0.75$




$$\Longrightarrow \ell \lesssim 22\text{km}$$

- Better by $10^7$ than Solar System bounds

# Future work

- $a \to GM$ limit analytically?
- All $a$ analytically?
- Rest of the metric
- Accretion disk modeling

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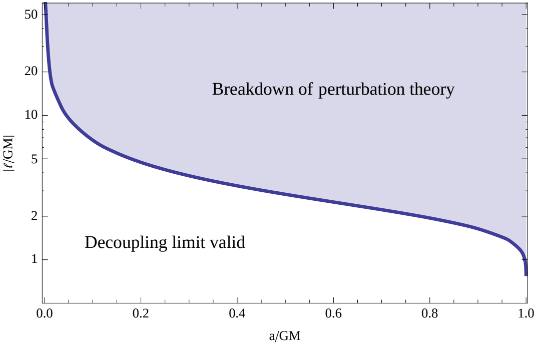


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