Implications of f(Q) Gravity on Cosmological Parameters

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Introduction

The accelerated expansion of the universe is one of the most fascinating and enigmatic phenomena in modern cosmology. While General Relativity (GR), combined with a cosmological constant, remains the standard model for explaining this acceleration, it still raises fundamental questions, particularly regarding the nature of dark energy and its role in cosmic dynamics.

f(Q) gravity offers an innovative alternative by reformulating the description of gravitational interactions. Unlike GR, which relies on spacetime curvature through the Ricci scalar R, f(Q) gravity introduces non-metricity, a geometric property that describes how vector lengths change under parallel transport in spacetime. This approach provides a geometric interpretation of dark energy and the accelerated expansion, eliminating the need for a cosmological constant.

In this study, we explore the implications of f(Q) gravity on key cosmological parameters by adopting the Friedmann-Lemaître-Robertson-Walker (FLRW) model, which describes a homogeneous and isotropic universe. By deriving the modified Friedmann equations, we analyze essential parameters, such as energy density, pressure, the equation of state parameter, the deceleration parameter, energy conditions (NEC, DEC, SEC), and the Hubble parameter as functions of redshift.This analysis aims to provide deeper insights into the dynamic evolution of the universe and to evaluate the viability of f(Q) gravity as a promising alternative to General Relativity for explaining cosmic acceleration.

Overview of the f(Q) gravity theory

The f(Q) gravity theory introduces a novel approach to gravitational interactions by modifying the traditional Einstein-Hilbert action. In this framework, the Ricci scalar R, used in General Relativity, is replaced with a function f(Q), where Q is the non-metricity scalar. This scalar Q encapsulates the geometric property of non-metricity, which describes the behavior of vector lengths during parallel transport. By incorporating non-metricity, f(Q) gravity allows for new interpretations of gravitational phenomena, including the potential to explain dark energy and the accelerated expansion of the universe.

The action for f(Q) gravity is given by:

$$S = \int \left[\frac{1}{2}f(Q) + \mathcal{L}_m\right] d^4x\sqrt{-g} \tag{1}$$

To obtain the modified field equations, we vary the action with respect to the metric $g_{\mu\nu}$:

$$\frac{2}{\sqrt{-g}}\nabla_\alpha(f_Q\sqrt{-g}P^\alpha_{\mu\nu}) + \frac{1}{2}fg_{\mu\nu} + f_Q(P_{\mu\alpha\beta}Q_\nu^{\alpha\beta} - 2Q_\mu^{\alpha\beta}P_{\alpha\beta\nu}) = -T_{\mu\nu} \tag{2}$$

Additionally, by varying the action with respect to the affine connection, we obtain a supplementary condition:

$$\nabla_\mu\nabla_\gamma\left(\sqrt{-g}f_Q P^{\gamma\mu\nu}\right) = 0. \tag{3}$$

Cosmological Model

To examine the impact of f(Q) gravity on cosmic evolution, we adopt the FLRW model, which describes a homogeneous and isotropic universe. This model is widely used for studying large-scale cosmic expansion and is based on the cosmological principle of uniformity and isotropy.

The metric of the FLRW model is given by the line element:

$$ds^2 = -dt^2 + a^2(t)[dx^2 + dy^2 + dz^] \tag{4}$$

In this study, we consider a linear form of the non-metricity scalar Q for simplicity:

$$f(Q) = \alpha Q + \beta \tag{5}$$

Using the above assumptions, we derive the modified Friedmann equations for the FLRW model within the f(Q) gravity framework. The classical Friedmann equations are modified to include the effects of non-metricity and are given by:

$$3H^2 = \frac{1}{2f_Q}(-\rho + \frac{f}{2}) \tag{6}$$

$$H + 3H^2 + \frac{f_Q}{f_Q}H = \frac{1}{2f_Q}(p + \frac{f}{2}) \tag{7}$$

where $H = \frac{a}{a}$ is the Hubble parameter, and dots denote derivatives with respect to cosmic time t. By combining these equations with the continuity equation for the cosmological fluid:

$$3H(\rho + p) = 0 \tag{8}$$

we obtain a complete set of differential equations describing the cosmological evolution within the FLRW model under f(Q) gravity.

Cosmological Parameter Analysis

In this study, we investigate how f(Q) gravity, an extension of General Relativity (GR), affects essential cosmological parameters. By deriving and analyzing the modified Friedmann equations, our findings suggest that f(Q) gravity provides a viable alternative to GR, particularly in explaining the universe's accelerated expansion. Below are the key parameters analyzed within the f(Q) framework.

•**Energy Density:**

$$\rho = 54\alpha H^4 - \frac{\beta}{2} \tag{9}$$

•**Pressure:**

$$p = \frac{\beta}{2} - 18\alpha[3 + 4\frac{H}{H^2}]H^4 \tag{10}$$

•**Equation of State Parameter:**

$$\omega = \frac{p}{\rho} = \frac{2\left(\frac{\beta}{2} - 18\alpha H^3(3H + 4\dot{H})\right)}{108\alpha H^4 - \beta} \tag{11}$$

•**Energy Conditions:**

$$\begin{aligned}
&\text{NEC(Null Energy Condition)} \Longleftrightarrow 54\alpha H^4 - 18\alpha H^4\left(\tfrac{4\dot{H}}{H^2} + 3\right)\\
&\text{DEC(Dominant Energy Condition)} \Longleftrightarrow 54\alpha H^4 - \left|\tfrac{\beta}{2} - 18H^4\left(\tfrac{4\dot{H}}{H^2} + 3\right)\alpha\right| - \tfrac{\beta}{2}\\
&\text{SEC(Strong Energy Condition)} \Longleftrightarrow 54\alpha H^4 + 3\left(\tfrac{\beta}{2} - 18\alpha H^4\left(\tfrac{4\dot{H}}{H^2} + 3\right)\right) - \tfrac{\beta}{2}
\end{aligned} \tag{12}$$

•**Deceleration Parameter:**

$$q(z) = -1 + \frac{1}{k}\left[\frac{1}{f(z) + 1}\right]^2 \tag{13}$$

•**Hubble Parameter:**

$$H(z) = \frac{H_0 b}{k + b}\left[\frac{1}{f(z)} + 1\right] \tag{14}$$

Results

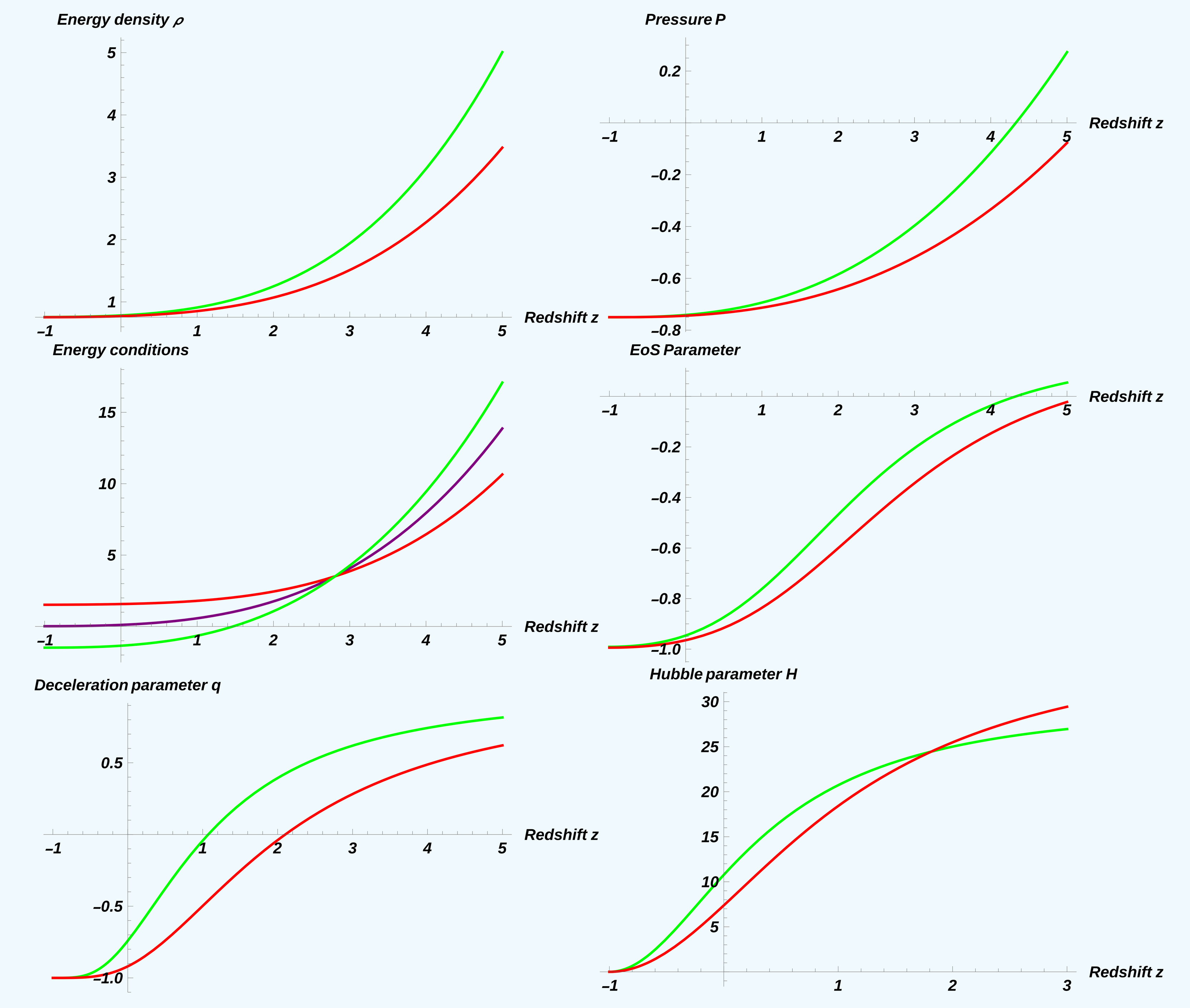

Figure 1. Evolution of Key Cosmological Parameters as a Function of Redshift z in the Context of f(Q) Gravity

Conclusion

This study provides a deeper understanding of how f(Q) gravity influences the evolution of key cosmological parameters and offers a fresh perspective on cosmic dynamics. By examining the modified Friedmann equations, we reveal the distinct behavior of parameters such as the Hubble rate, energy density, pressure, and equation of state within the f(Q) framework.

Our results highlight f(Q) gravity as a compelling alternative to General Relativity, particularly in explaining the universe's accelerated expansion without relying on a cosmological constant. This approach opens new pathways for addressing persistent challenges in cosmology and suggests that non-metricity-based gravity models could offer viable solutions.

Future work may build upon these findings by investigating more complex f(Q) models and their influence on other cosmological phenomena. This research contributes to the ongoing effort to develop a comprehensive gravitational theory that aligns with both current observations and theoretical advancements.

References

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