Pile-up Fractions and Count Rates

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Photon pile-up occurs when two or more photons interact with the same detection cell within the same CCD frame[1]. The CCD will be unable to resolve the individual photon events and instead record a single event with a PHA that is roughly the sum of the individual event PHAs. Hence pile-up results in a reduction of the observed count rate and a PHA spectrum that is skewed towards higher energies.

The purpose of this memo is to outline a simple procedure for estimating the amount of pile-up in an observation based upon the count-rate. It must be stated from the outset that this estimate is a *rough* one and is meant more as an indicator for pile-up.

Consider a Poisson-distributed flux of photons incident upon the CCD. Only a certain fraction of these photons will interact with CCD to produce a measurable charge cloud. Let r represent the number of such charge-cloud producing photons per frame incident upon a CCD detection cell. In this simplified treatment, the energy-dependence of the rate will be ignored. From basic Poisson statistics it follows that the probability for 2 or more photons to each produce charge-clouds in the region during a frame is given by

$$e^{-r}\sum_{k=2}^{\infty}\frac{r^k}{k!}. \tag{1}$$

The *pile-up fraction* in the region represents the fraction of event producing charge-clouds that are due to pile-up. It is given by

$$f=\sum_{k=2}^{\infty}\frac{r^k}{k!}\bigg/\sum_{k=1}^{\infty}\frac{r^k}{k!}, \tag{2}$$

or more simply as

$$f=\frac{1-(1+r)e^{-r}}{1-e^{-r}} \tag{3}$$

$$=\frac{1}{2}r-\frac{1}{12}r^2+O(r^4). \tag{4}$$

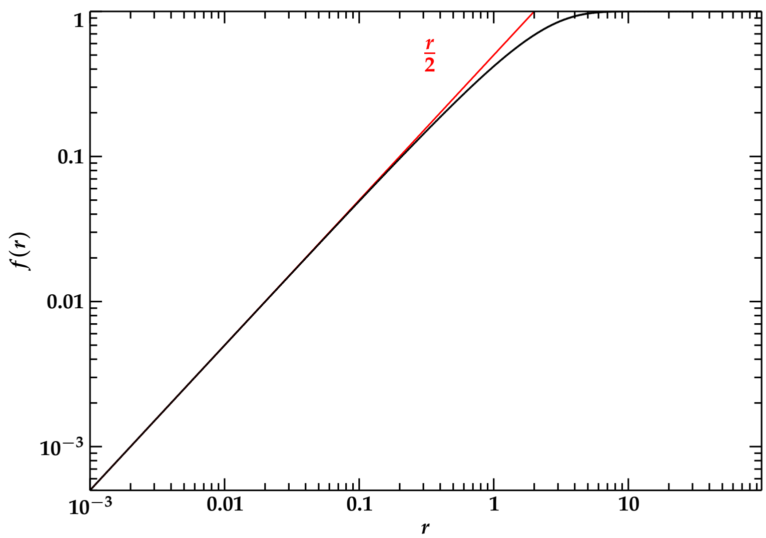

Figure 1: A plot of equation 3 and its linear approximation.

A plot of $f(r)$ as a function of r is shown Figure 1. As can been seen from the plot, the pile-up fraction approaches 1 with increasing r. The red curve represents the linear term of equation 4. In particular, this plot shows that the linear approximation is good for pile-up fractions up to 10 percent.

The quantity r can be related to the *observed* count-rate for "good-grade" events as follows. Following [1], let g_0 represent the probability that a charge cloud generated by a single photon will correspond to a good grade. Also, let α represent the (suitably averaged) probability that two such charge clouds will produce an event with a good grade. Then the the probability p that a good event will be generated in the detection cell is given by

$$p = e^{-g_0 r} \sum_{k=1}^{\infty} \alpha^{k-1} \frac{(g_0 r)^k}{k!}. \tag{5}$$

For an observation consisting of N frames, pN represents the expected number of good-grade events during the observation in the detection cell. Hence p may be regarded as detection cell's expected count-rate for good-grade events per frame.

To second order the above equation may be written

$$p = g_0 r - (1 - \frac{\alpha}{2})(g_0 r)^2 + O(r^3), \tag{6}$$

which shows explicitly that the effect of pile-up is to reduce the count-rate with increasing flux. Equation 3 may be solved for r in terms of the pile-up fraction to yield

$$r = 2f + \frac{2}{3} f^2 + O(f^3). \tag{7}$$

As indicated above, for pile-up fractions up to 10 percent, the first-order approximation may be used. It is easily seen that to first order, the count rate p is related to the pile-up fraction by

$$p = 2g_0 f + O(f^2). \tag{8}$$

The branching ratio g_0 is a function of energy and lies somewhere close to 1 except at high energies (above a few keV) where it becomes smaller. In the following, g_0 will be taken to be 1. With this assumption, it is easy to construct the following table showing the expected detection-cell count-rates for various pile-up fractions:

Pileup Fraction	**Count rate per frame**	**Count rate per sec**
1%	0.02	0.007
5%	0.1	0.03
10%	0.2	0.07

The last column of the table assumes a standard frame-time of 3.2 seconds. It must be emphasized that these numbers area meant to serve only as guide and that the rates pertain to a detection cell, which for ACIS is a 3x3 pixel region.

References

[1] Davis, J.E., 2001, ApJ, 562, 575