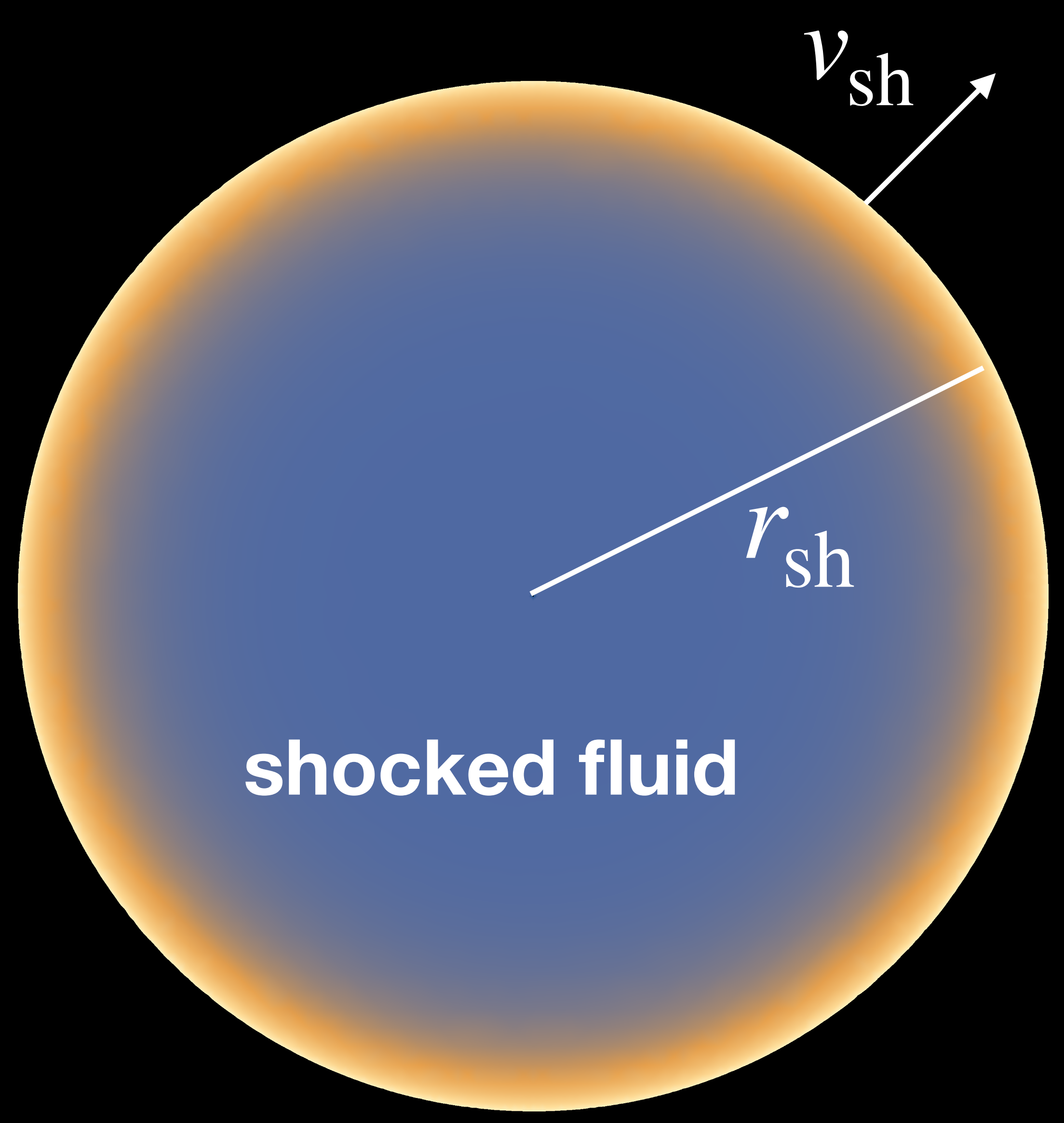
Self-similar, Weak Shock Propagation with Accretion

Eric Coughlin
Columbia University
arxiv:1805.06456 (2018 ApJ, 263, 158)
Collaborators: Eliot Quataert, Stephen Ro (Berkeley)

Outline

- Self-similarity?
  - Sedov-Taylor blastwave
- New self-similar solutions
- Application to failed supernovae
- Summary/Conclusions
- Questions

Self-similarity



- Mass, momentum, energy must be conserved across this shock. Yields jump conditions:

$$v[r_{\rm sh}(t)] \propto v_{\rm sh}, \quad \rho[r_{\rm sh}(t)] \propto \rho_{\rm a}, \quad p[r_{\rm sh}(t)] \propto \rho_{\rm a} v_{\rm sh}^2$$

- These boundary conditions occur at a time-dependent location, but can remove this if we introduce $\xi \equiv r/r_{\rm sh}(t)$ in place of Eulerian radius

- Assume solutions to fluid equations of the form $v = v_{\rm sh}(t)\, f(\xi), \quad \rho = \rho_{\rm a}\, g(\xi), \quad p = \rho_{\rm a}\, v_{\rm sh}(t)^2\, h(\xi)$

- This works if $r_{\rm sh}^{\alpha}\, v_{\rm sh}^2 = \text{const}$

- If energy conserved, $E = \int_0^{r_{sh}} \frac{1}{2}\rho v^2 r^2 dr = \frac{1}{2} r_{sh}(t)^3 v_{sh}(t)^2 \rho_a \int_0^1 \xi^2 f(\xi) g(\xi) d\xi \quad \Leftrightarrow \quad \alpha = 3$ $\Rightarrow r_{\rm sh} \propto t^{2/5}$

New Self-similar Solutions

- Recall assumptions of ST:
  - Strong shock — neglect ambient sound speed
    - Kinetic energy >> Thermal energy of ambient gas
  - No gravity
    - Kinetic energy >> Grav energy
- What happens when these are not satisfied?
  - Does Sedov-Taylor (~ energy-conserving) still describe shock propagation?

New Self-similar Solutions

- In general, probably not:
    - In grav. field, sweeping up ambient material adds binding energy
        - Total energy behind shock not conserved
    - Jump conditions depend on Mach number
        - Adds additional spatial dependence
    - Gravity adds additional timescale/lengthscale

New Self-similar Solutions

- But:
  - If ambient gas in HSE with *point mass*, and adiabatic: $\rho = r^{-n}, \quad p = \frac{1}{n+1}\frac{GM}{r}r^{-n}, \quad c_s \simeq \sqrt{\frac{GM}{r}}$
  - If kinetic energy ~ grav potential energy, $\frac{1}{2}v^2 \simeq \frac{GM}{r} \quad \Rightarrow \quad v \simeq \sqrt{\frac{GM}{r}}$
  - And if we let

$$v_{\rm sh} = V\sqrt{\frac{GM}{r_{\rm sh}}} \quad \Rightarrow \quad r_{\rm sh} = \left(\frac{3}{2}V\sqrt{GM}t\right)^{2/3}, \quad v = v_{\rm sh}f(\xi), \quad \rho = \rho_a(r_{\rm sh})g(\xi), \quad p = \rho_a(r_{\rm sh})v_{\rm sh}^2 h(\xi), \quad \xi = \frac{r}{r_{\rm sh}(t)}$$

  - Then
    - Mach ~ constant, boundary conditions satisfied self-similarly
    - Inserting above into fluid equations gives three ODEs
    - Can numerically integrate to find solutions; importantly depend on V!

New Self-similar Solutions

- What sets shock velocity V?
  - For Sedov-Taylor, V determined from energy constraint
  - Here, however, energy behind shock not conserved…
- Can show that there is a sonic point in these self-sim sols
  - In order for quantities to smoothly pass through, need special value of V

Application to Failed Supernovae

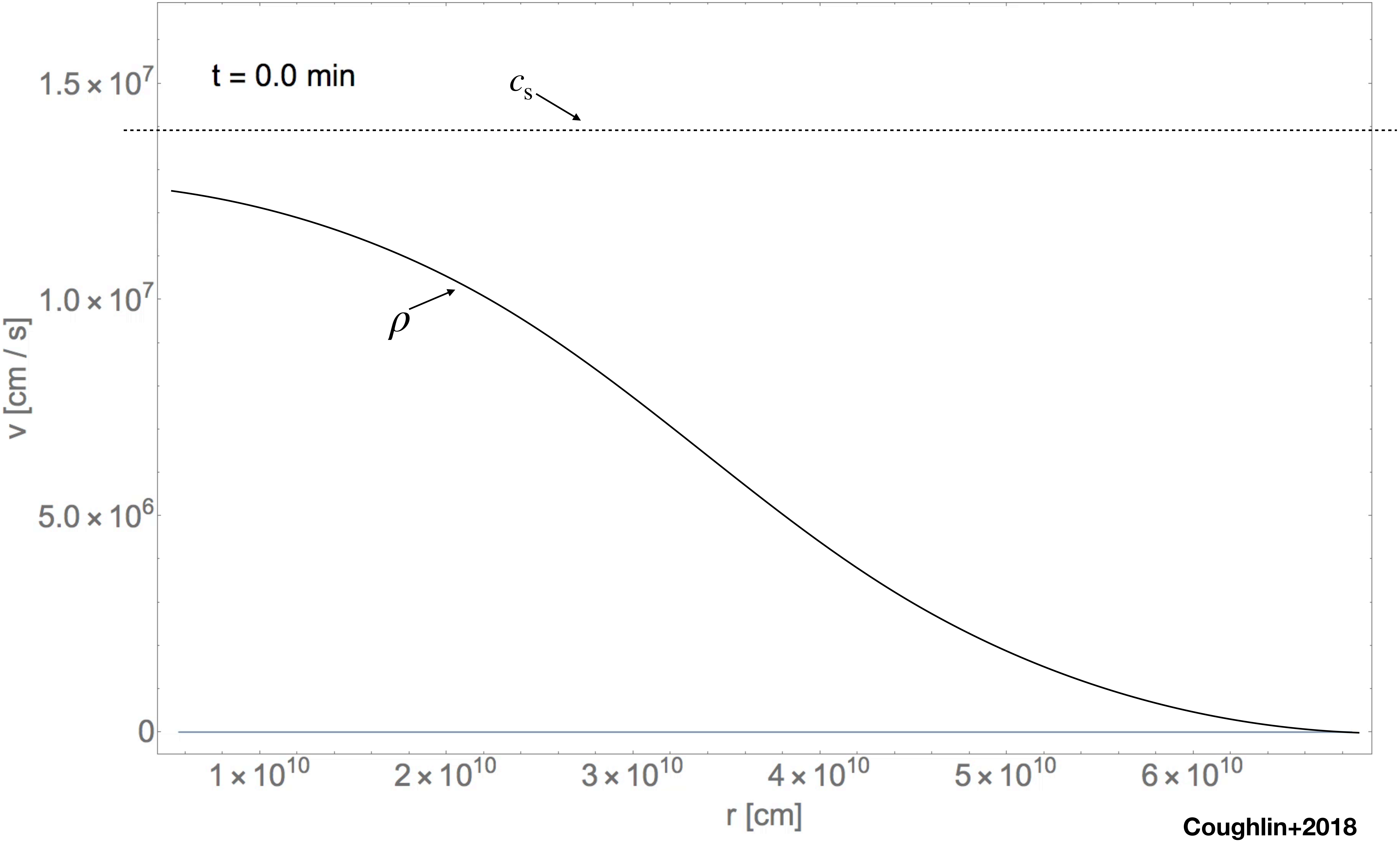
- One potential application = failed supernova:
  - Massive star, core collapse
  - Protoneutron star forms, bounces, launches shock
  - If shock stalls and cannot be revived:
    - Continued accretion forms black hole
    - Star is accreted by black hole
    - Disappearing star…

Application to Failed Supernovae

- …But that's not all:
  - During neutron star formation
    - Ton of neutrinos radiated, $\sim few \times 0.1 M_{\odot}$
    - Reduces gravitational field

Application to Failed Supernovae

- While stalled shock is sitting there
  - Overlying envelope (still in ~ HSE) responds to changing gravitational field
  - Result: weak sound pulse generated in interior of star
    - ⋆ Nadyozhin 1980; Lovegrove & Woosley 2013
  - Pulse steepens as it goes down density gradient



t = 0.0 min
c_s
ρ
1.5×10^7
1.0×10^7
5.0×10^6
0
v [cm / s]
1×10^{10}
2×10^{10}
3×10^{10}
4×10^{10}
5×10^{10}
6×10^{10}
r [cm]
Coughlin+2018

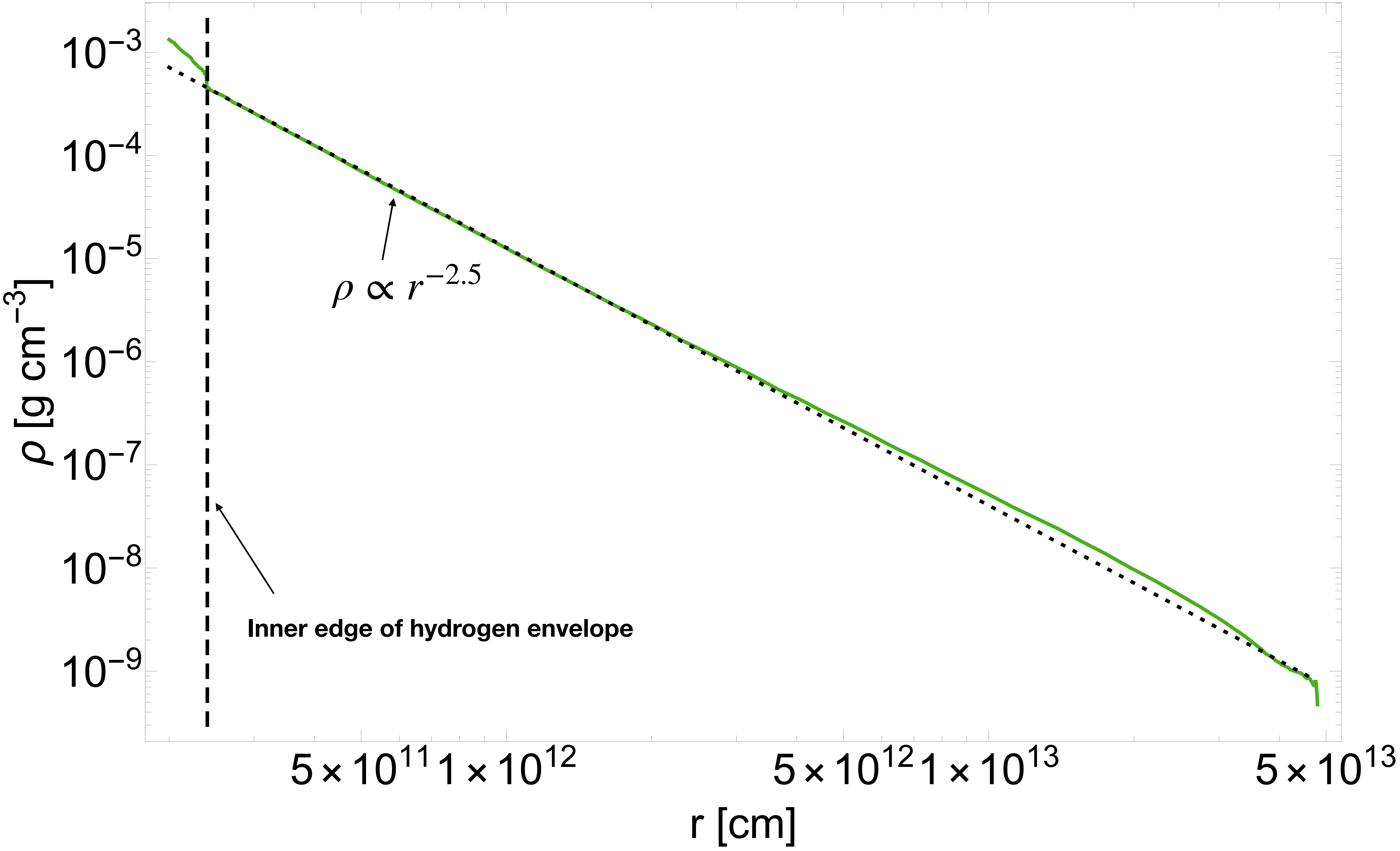
Failed Supernovae

- For supergiant:
  - Pulse steepens into weak shock (Mach ~ 1) near base of hydrogen envelope
  - Shock propagates through ~2-3 decades in radius
- Importantly, this all happens while the shock stalls, then fails, then creates a black hole and results in accretion
- Thus, have outward-propagating ~ weak shock, accretion at center, so conditions seem right…

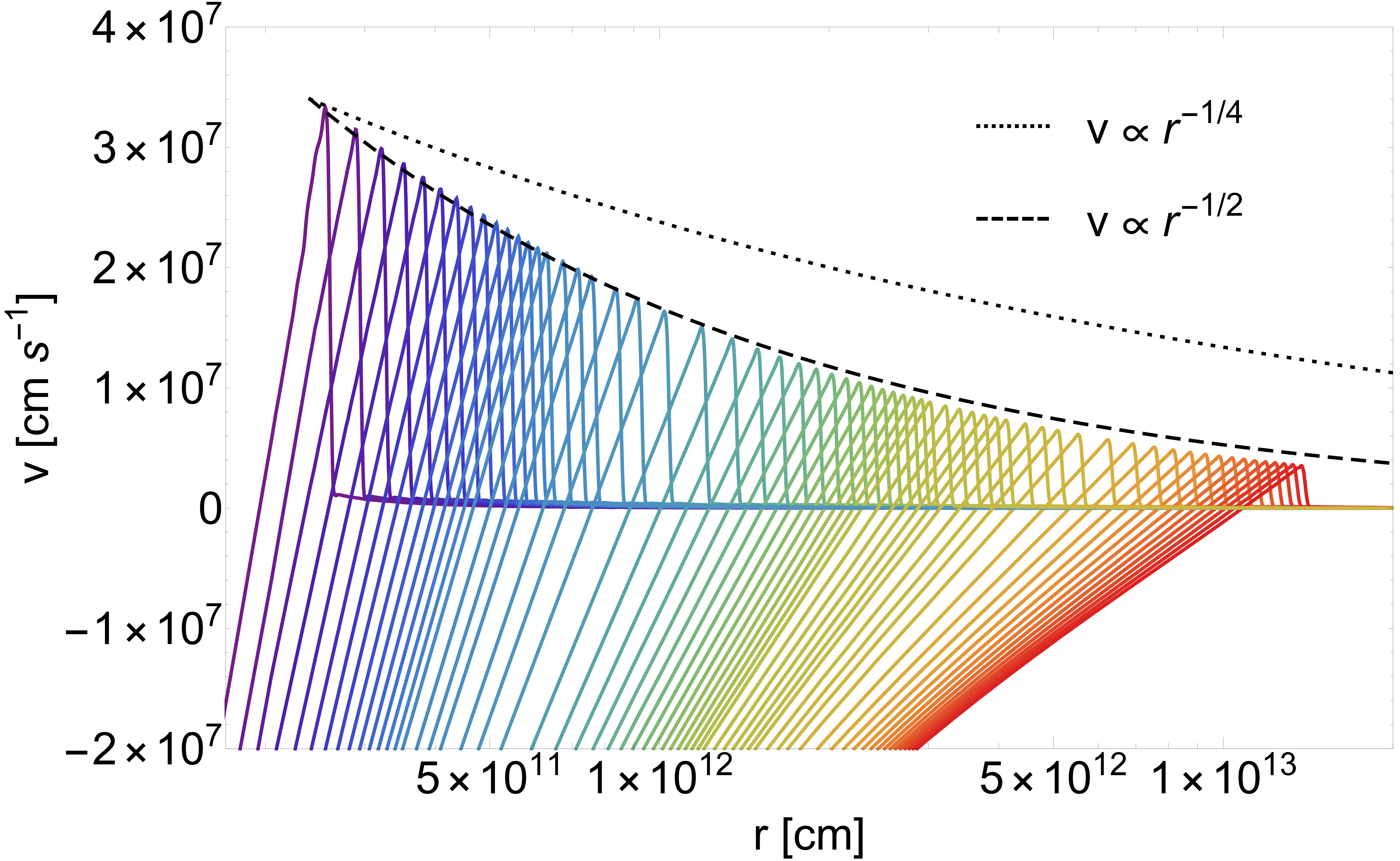
Application to Failed Supernovae

- Fernandez+(2018) ran simulations (FLASH) of failed supernovae for RSGs, BSGs, YSGs, WRs (MESA)
- Focus on specific case of $22\,M_{\odot}\mathrm{YSG}$
- Why?

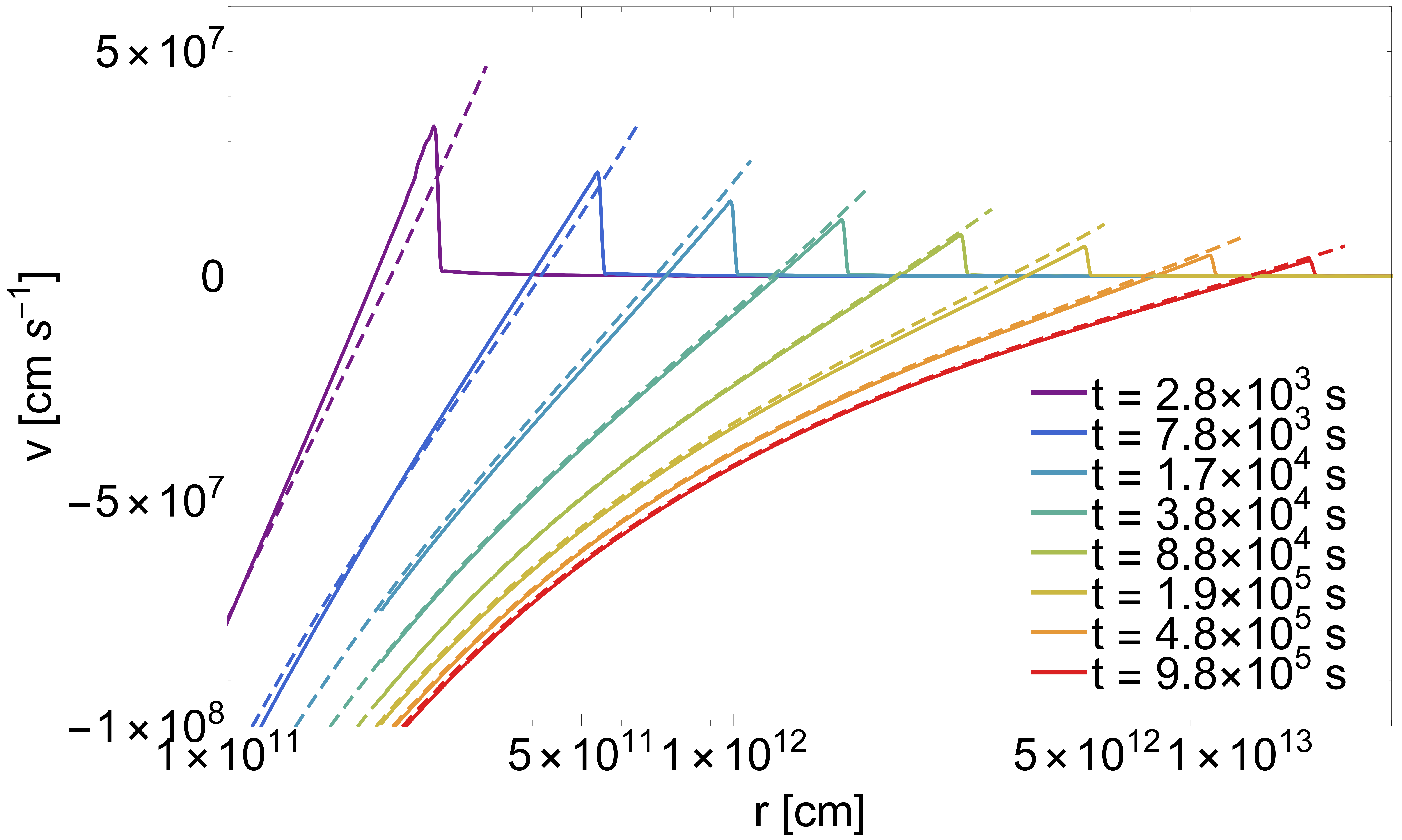
Eric Coughin, eric.r.coughlin@gmail.com



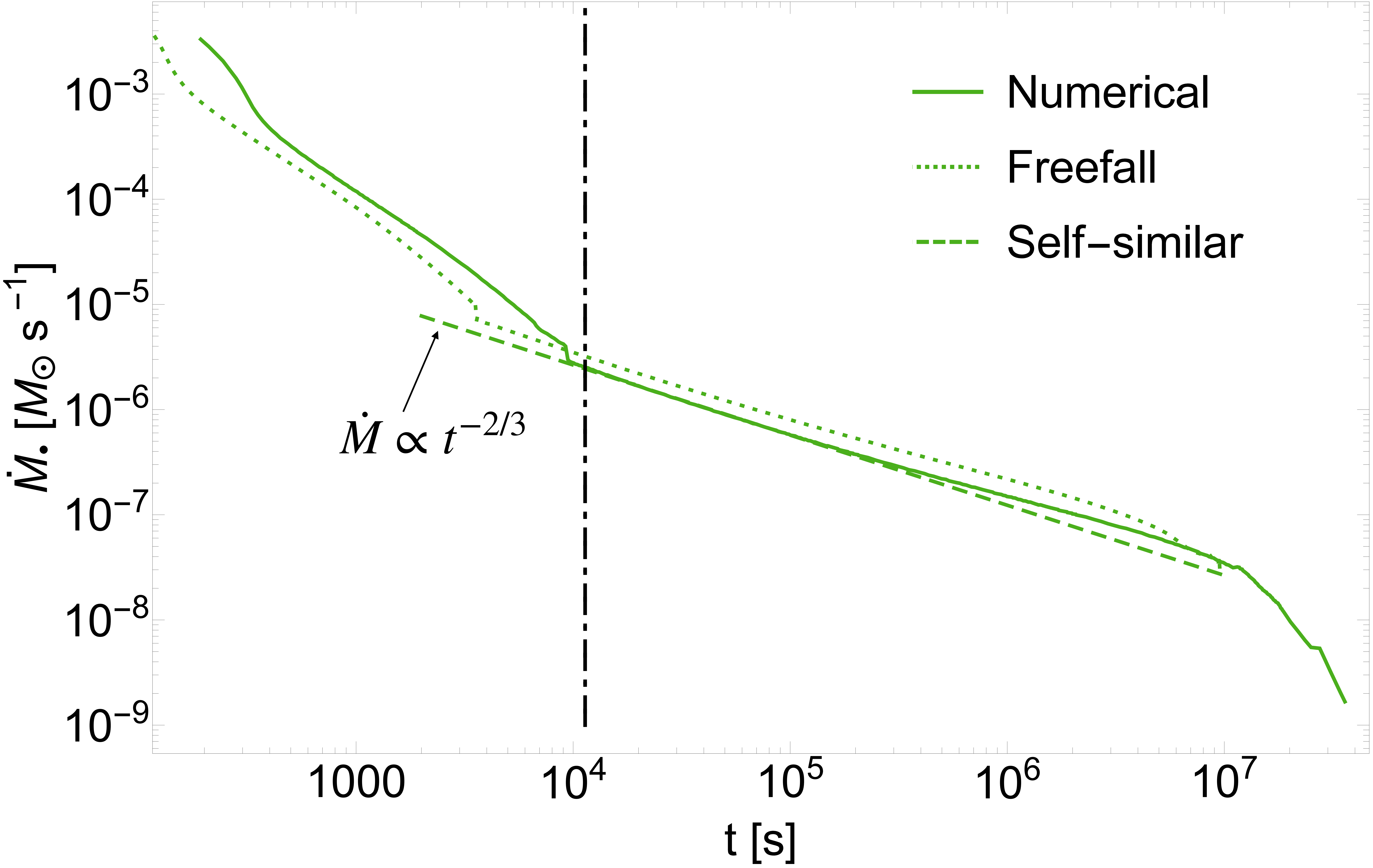
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
ρ [g cm^{-3}]
$\rho \propto r^{-2.5}$
Inner edge of hydrogen envelope
5×10^{11}
1×10^{12}
5×10^{12}
1×10^{13}
5×10^{13}
r [cm]



4 × 10^7
3 × 10^7
2 × 10^7
1 × 10^7
0
−1 × 10^7
−2 × 10^7
v [cm s^−1]
5 × 10^11
1 × 10^12
5 × 10^12
1 × 10^13
r [cm]
v ∝ r^−1/4
v ∝ r^−1/2



5×10^7
0
−5×10^7
−1×10^8
v [cm s^−1]
1×10^11
5×10^11
1×10^12
5×10^12
1×10^13
r [cm]
t = 2.8×10^3 s
t = 7.8×10^3 s
t = 1.7×10^4 s
t = 3.8×10^4 s
t = 8.8×10^4 s
t = 1.9×10^5 s
t = 4.8×10^5 s
t = 9.8×10^5 s



Numerical
Freefall
Self–similar
$\dot{M} \propto t^{-2/3}$
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
1000
10^{4}
10^{5}
10^{6}
10^{7}
t [s]
$\dot{M}_{\bullet}$ [$M_{\odot}$ s^{-1}]

Conclusions

- Actually seems to work!
  - Predicts propagation of shock
  - Predicts time and space-dependent velocity, density, pressure
  - Predicts accretion rate
  - Predicts amount of ejecta
- If we see one of these things (and the accretion generates a luminous outburst)
  - Maybe we can use these to predict stellar properties?
  - Black hole properties?